📘 Class 10 Mathematics – Complete Notes

Chapter 1: Real Numbers

🔹 Summary

- Euclid's Division Lemma: किसी भी दो धनात्मक पूर्णांक a, b के लिए, $a = bq + r$, जहाँ $0 \leq r < b$
- Fundamental Theorem of Arithmetic: हर संख्या का unique prime factorization होता है।
- HCF और LCM prime factorisation या Euclid's Algorithm से निकाले जा सकते हैं।

🔹 Formulas

- $HCF \times LCM = \text{Product of numbers}$

Example

Find HCF of 56 and 72.

- $56 = 2^3 \times 7, 72 = 2^3 \times 3^2$
- HCF = $2^3 = 8$

Important Questions

1. Find LCM and HCF of 26 and 91 by prime factorization.
2. Show that √2 is an irrational number.

Chapter 2: Polynomials

Summary

- Linear polynomial: degree 1
- Quadratic polynomial: degree 2
- Cubic polynomial: degree 3

- Relationship between zeros and coefficients:

Formulas

- For quadratic $ax^2 + bx + c$:
  $\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$

Example

Find sum and product of zeros of $2x^2 - 5x + 3$.

- Sum = $-\frac{-5}{2} = \frac{5}{2}$
- Product = $\frac{3}{2}$

Important Questions

1. If α and β are zeros of $6x^2 - 7x - 3$, find relation.
2. Form a quadratic polynomial whose sum and product of zeros are -1 and 4.

Chapter 3: Pair of Linear Equations

Summary

- General form: $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$
- Graphical and algebraic methods available.

Condition of solutions

- Unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
- Infinite solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- No solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Example

Solve by substitution:
$2x + y = 5,; 3x - y = 5$
→ Add: $5x = 10$, so $x = 2, y = 1$

Important Questions

1. Solve using elimination:
   $x + 2y = 7, 2x - 3y = 1.$
2. A boat goes 16 km downstream in 2 hrs and returns upstream in 4 hrs. Find speed of boat in still water and speed of stream.

Chapter 4: Quadratic Equations

Summary

- Standard form: $ax^2 + bx + c = 0$
- Solution methods: Factorization, completing square, quadratic formula.

Formula

- Roots: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant $D = b^2 - 4ac$ decides

Example

Solve: $x^2 - 7x + 12 = 0$
→ Factorization: $(x - 3)(x - 4) = 0$ →
Roots = 3, 4

Important Questions

1. Solve $2x^2 - 7x + 3 = 0$.
2. Find quadratic equation whose roots are 2 and -3.

Chapter 5: Arithmetic Progression

Summary

- Sequence with constant difference.

Formulas

- $a_n = a + (n - 1)d$

- $S_n = \frac{n}{2}[2a + (n-1)d]$

Example

AP: 2, 7, 12,... Find 10th term.

- $a = 2, d = 5 \rightarrow a_{10} = 2 + (9)(5) = 47$

Important Questions

1. Find sum of first 25 multiples of 8.
2. Which term of AP 3, 8, 13, ... is 78?

Chapter 6: Triangles

Summary

- Similarity rules: AA, SAS, SSS.
- Areas ratio = square of corresponding sides ratio.

Theorems

- Pythagoras Theorem: $a^2 + b^2 = c^2$

Example

In ΔABC, DE || BC, AD=3, DB=2. Find AE/EC.
→ By Basic Proportionality Theorem,
$\frac{AD}{DB} = \frac{AE}{EC} \implies AE/EC = 3/2$

Important Questions

1. Prove: In a right triangle, square on hypotenuse = sum of squares on other two sides.
2. In ΔABC, AD ⊥ BC, prove $AB^2 = BD \times BC$.

Chapter 7: Coordinate Geometry

Summary

- किसी भी दो points के बीच distance, midpoint, section point find करने के लिए formulas।
- Useful in proving collinearity और finding centroid, coordinates इत्यादि।

Formulas

- Distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Example

Find distance between A(2,3) and B(10,6).

$$d = \sqrt{(10-2)^2 + (6-3)^2} = \sqrt{64+9} = \sqrt{73}$$

Important Questions

1. Find coordinates of the point dividing (2,3) and (10,6) in ratio 1:3.
2. If (1,2), (4,y), (x,6) are collinear, find x and y.

Chapter 8: Introduction to Trigonometry

Summary

- Trigonometric ratios of acute angle defined using right triangle.
- Relation between ratios → identities.

Ratios

$\sin\theta = \frac{P}{H}$, $\cos\theta = \frac{B}{H}$, $\tan\theta = \frac{P}{B}$, $\csc\theta = \frac{H}{P}$, $\sec\theta = \frac{H}{B}$, $\cot\theta = \frac{B}{P}$

Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$

Example

If $\sin\theta = 3/5$, find $\cos\theta$.
$\rightarrow P = 3, H = 5, B = \sqrt{25 - 9} = 4$
$\cos\theta = 4/5$.

Important Questions

1. Prove: $\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$.
2. If $\cos A = 12/13$, find other trigonometric ratios.

Chapter 9: Applications of

Summary

- Heights and distances problems using trigonometric ratios.
- Angle of elevation: ऊपर देखने का कोण।
- Angle of depression: नीचे देखने का कोण।

Example

A 15 m high tower casts a shadow 20 m long. Find angle of elevation of sun.
$\tan\theta = 15/20 = 3/4$.
So $\theta = \tan^{-1}(0.75)$.

Important Questions

1. A balloon is observed at an angle of elevation 45° at height 50 m. Find distance of observer from foot of balloon.
2. A man is standing 40 m away from building. Angle of elevation of top is 30°. Find height of building.

Chapter 10: Circles

Summary

- Tangent: line touching circle at one point.
- Properties:
  - Tangent ⊥ radius at point of contact.
  - From an external point, two tangents can be drawn and they are equal.

Example

Two tangents are drawn to a circle of radius 5 cm from an external point whose distance is 13 cm from centre. Find length of tangent.
By Pythagoras:
$x^2 + 5^2 = 13^2 \implies x^2 = 169 - 25 = 144 \implies x = 12$

.

Important Questions

1. Prove: Tangents drawn from external point are equal.
2. From point 10 cm away from centre of circle radius 6 cm, find length of tangent.

Chapter 11: Constructions

Summary

- Division of line segment in ratio.
- Tangents to circle from external point.
- Constructing similar triangles.

Important Questions

1. Divide a line segment of 9 cm in ratio 4:5.

2. Draw tangents to circle from a point 6 cm away from centre of circle radius 3 cm.

Chapter 12: Areas Related to Circles

Formulas

- Area of circle: πr^2
- Area of sector: $\frac{\theta}{360}\pi r^2$
- Length of arc: $\frac{\theta}{360} \times 2\pi r$

Example

Find area of sector of angle 60° in circle of radius 7 cm.
$\frac{60}{360}\pi(7^2) = \frac{1}{6}\pi \times 49 = \frac{49\pi}{6}$

Important Questions

1. Find area of segment of circle radius 14 cm, angle 90°.
2. Find length of arc subtending angle 120° at centre of circle radius 21 cm.

Chapter 13: Surface Areas and Volumes

Formulas

- **Cube:** TSA = $6a^2$, Vol = a^3
- **Cuboid:** TSA = $2(lb + bh + hl)$, Vol = lbh
- **Sphere:** TSA = $4\pi r^2$, Vol = $\frac{4}{3}\pi r^3$
- **Cylinder:** CSA = $2\pi rh$, Vol = $\pi r^2 h$
- **Cone:** CSA = πrl, Vol = $\frac{1}{3}\pi r^2 h$
- **Hemisphere:** CSA = $2\pi r^2$, Vol = $\frac{2}{3}\pi r^3$

Example

Find volume of cone of radius 7 cm, height 24 cm.
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(49)(24) = 1232\pi$

Important Questions

1. Find volume of sphere of radius 7 cm.
2. A solid metallic cylinder of height 15 cm and radius 7 cm is melted and recast into spheres. Find number of spheres radius 3.5 cm.

Chapter 14: Statistics

Summary

- Mean, Median, Mode from grouped data.

Formulas

- **Mean:** $\bar{x} = \frac{\Sigma f x}{\Sigma f}$

- **Median:**
  $L + \frac{(N/2 - cf)}{f} \times h$
- **Mode:**
  $L + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$

Example

Find mean of following data:

x: 1, 2, 3, 4
f: 2, 3, 4, 1

$\bar{x} = \frac{1\times2 + 2\times3 + 3\times4 + 4\times1}{10} = \frac{2+6+12+4}{10} = 24/10 = 2.4$

Important Questions

1. Find mode of data:
   Class 0–10: 5, 10–20: 8, 20–30: 10, 30–40: 12, 40–50: 7.
2. Find median from data:
   Class 0–10: 10, 10–20: 20, 20–30: 15, 30–40: 25, 40–50: 20.

Chapter 15: Probability

Summary

- Probability = chance of event happening.
- □ P(E)=\frac{\text{Number of favourable outcomes}}{\text{Total outcomes}} □

Example

Find probability of getting head in coin toss.
$P = \frac{1}{2}$.

Important Questions

1. A card is drawn from pack of 52. Find probability of getting (i) red card (ii) face card.
2. Two dice are thrown. Find probability of getting sum = 7.